Mat 2377

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Solution 2 on (30)

2. 2 $(2points)$

S	x
NNN	0
NNB	1
NBN	1
BNN	1
NBB	2
BNB	2
BBN	2
BBB	3

2.6 $(2points)$a) $P(X > 200) = \int_{200}^{\infty} \frac{20000}{(x+100)^3} dx = \frac{1}{9}$

b)$P(80 < X < 200) = \int_{80}^{200} \frac{20000}{(x+100)^3} dx = \frac{1000}{9801} = 0.102$

2.10 $(4points)$

(a) $P(T = 5) = F(5) - F(4) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$

(b) $P(T > 3) = 1 - F(3) = 1 - \frac{1}{2} = \frac{1}{2}$

(c) $P(1.4 < T < 6) = F(6) - F(1.4) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

d) $P(T \leq 5 | T \geq 2) = \frac{P(2 \leq T \leq 5)}{P(T \geq 2)} = \frac{\frac{3}{4} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{2}{3}$

2.22 $(6points)$

(a) $1 = k \int_{-1}^{1} (3 - x^2) dx = k \left(3x - \frac{x^3}{3}\right) |_{-1}^{1} = \frac{16}{3} k$. Hence $k = \frac{3}{16}$

(b) $P\left(X < \frac{1}{2}\right) = \frac{3}{16} \int_{-1}^{\frac{1}{2}} (3 - x^2) dx = \frac{3}{16} \left(3x - \frac{x^3}{3}\right) |_{-1}^{\frac{1}{2}} = \frac{99}{128}$

c) Note $F(t) = \frac{1}{2} + \frac{9}{16}t - \frac{t^3}{16}, -1 \le t < 1$

$$\begin{aligned} P(|X| < 0.8) &= P((X < -0.8)) + P(X > 0.8) \\ &= F(-0.8) + 1 - F(0.8) \\ &= 0.164 \end{aligned}$$

2.30 ($6points$) The joint pmf is given in the table

			x		
f(x, y)		0	1	2	3
	0	0	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$
y	1	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$
	2	$\frac{2}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{5}{0}$

a) $P(X \le 2, Y = 1) = f(0,1) + f(1,1) + f(2,1) = \frac{6}{30}$
b) $P(X > 2, Y \le 1) = f(3,0) + f(3,1) = \frac{7}{30}$
c)

$$\begin{aligned} P(X > Y) &= f(1,0) + f(2,0) + f(3,0) + f(2,1) + f(3,1) + f(3,2) \\ &= \frac{18}{30} \end{aligned}$$

d) $P(X + Y = 4) = f(2,2) + f(3,1) = \frac{8}{30}$

e)

x	0	1	2	3	y	0	1	2
$f_X(x)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{4}{10}$	$f_Y(y)$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{7}{15}$

2.34 ($2points$)Since the joint density is the product of a function of x and a separate function of y, the variables are independent. The marginal density of X is $f_X(x) = e^{-x}, x > 0$. Hence

$$P(0 < X < 1) = \int_0^1 e^{-x} dx = 0.6321$$

2.40 ($2points$)a)

x	2	4
$f_X(x)$	0.40	0.60

b)

y	1	3	5
$f_Y(y)$	0.25	0.50	0.25

2.58 ($2points$)

$$\begin{aligned} \mu_X &= \sum x f_X(x) = 1(0.17) + 2(0.5) + 3(0.33) = 2.16 \\ \mu_Y &= \sum y f_Y(y) = 1(0.23) + 2(0.5) + 3(0.27) = 2.04 \end{aligned}$$

2.78 $(2points)$

$$\begin{aligned} \mu_X &= \sum x f_X(x) = 0(0.4) + 1(0.3) + 2(0.2) + 3(0.1) = 1.0 \\ E\left[X^2\right] &= \sum x^2 f_X(x) = 1(0.3) + 4(0.2) + 9(0.1) = 2.0 \end{aligned}$$

Hence, $\sigma^2 = 2 - 1^2 = 1$
2.120 $(2points)$ let X be the number of components operating.
$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$

$$\begin{aligned} &= \binom{5}{3}(0.92)^3(1-0.92)^2 + \binom{5}{4}(0.92)^4(1-0.92) + \binom{5}{5}(0.92)^5 \\ &= 0.9955 \end{aligned}$$